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Sir W. R. Hamilton, having been lately induced to consider, in connexion with the Calculus of Quaternions, the celebrated theorem of Dupin, respecting the character of the intersection lines of three systems of orthogonal surfaces, as lines of curvature thereon, stated that he had thus been led to perceive some symbolical results which he supposed to be new, and which seemed to him to be of sufficient interest to be submitted to the Academy.

As long ago as 1846, he had proposed the notation,*

$$\triangleleft = i \frac{d}{dx} + j \frac{d}{dy} + k \frac{d}{dz};$$

and had pointed out a theorem,† differing only slightly in its expression from the following :

$$V . a V . \beta \gamma = \gamma S . a \beta - \beta S . a \gamma;$$

which may also be thus written,

$$V . a (V . \beta \gamma) = S . a \beta . \gamma - \beta S . a \gamma,$$

or thus,

$$V . a (V . \beta \gamma) = S . \beta a . \gamma - \beta S . a \gamma.$$

The recent results just referred to have a remarkable symbolical resemblance to those comparatively old ones, since they admit of being written thus :

$$\text{I.} \dots V . a (V . \triangleleft \nu) = S . a \triangleleft . \nu - \triangleleft S . a \nu;$$

$$\text{II.} \dots V . \triangleleft (V . \beta \nu) = S . \beta \triangleleft . \nu - \beta S . \triangleleft \nu;$$

where $\triangleleft$ is *not* an ordinary *vector*, but a certain *symbol of operation*, analogous to a *vector*, in its combinations with other symbols, and defined by a foregoing formula: while a and β are constant vectors, and ν is a variable vector, regarded as a

* See Proceedings of the Academy for July, 1846.

† See Philosophical Magazine for August, 1846.

function of xyz , or of $\rho = ix + jy + kz$, and subject as such to the operations $\triangleleft$, $S \cdot a \triangleleft$, $S \cdot \beta \triangleleft$; where

$$S \cdot a \triangleleft = - \left(a \frac{d}{dx} + b \frac{d}{dy} + c \frac{d}{dz} \right),$$

if $a = ia + jb + kc$, and the symbol $S \cdot \beta \triangleleft$ is similarly interpreted.

These were among the chief elements of calculation employed, in proving by quaternions the theorem above mentioned of Dupin; of which one expression, in the quaternion calculus, is the following:—

“If the three differential equations,

$$S \cdot v d\rho = 0, \quad S \cdot v' d\rho = 0, \quad S \cdot vv' d\rho = 0,$$

be integrable, and if $S \cdot vv' = 0$, then the supposition $V \cdot v' d\rho = 0$ conducts to the equation $S \cdot vv' dv = 0$.”

Another expression of the same theorem is as follows :

$$\text{“If} \quad S \cdot v \triangleleft v = 0, \quad S \cdot v' \triangleleft v' = 0, \quad S \cdot v'' \triangleleft v'' = 0,$$

$$\text{and} \quad V \cdot vv' v'' = 0,$$

$$\text{then} \quad S \cdot v'' (S \cdot v' \triangleleft v) = 0.”$$

In this last formula, the symbol $S \cdot v' \triangleleft v$ denotes a vector having the direction of $-dv$, if $d\rho$ have the direction of v' ; and the equation expresses, that if we thus move a little along the first surface in the direction of the normal to the second surface, the new or near normal to that first surface will be contained in the tangent plane to the third surface, and therefore will intersect the old normal to the first surface: which is a form of the theorem of Dupin.

Although not very closely connected with that well-known theorem, Sir W. R. H. wishes to add that another old form of his, for any three vectors, namely,

$$V(V \cdot \gamma \beta \cdot a) = \gamma S \cdot \beta a - \beta S \cdot \gamma a,$$

has suggested to him this new symbolical result,

$$\text{III.} \dots \quad V(V \cdot \gamma \triangleleft v) = \gamma S \cdot \triangleleft v - \triangleleft S \cdot \gamma v;$$

and that each of the three general theorems, expressed by the formulæ I. II. III. of this Abstract, can be proved to continue to be true, when his old signification of the symbol Δ , to which Mr. Carmichael's researches have lately given an additional interest, is changed to this other and more extensive signification,

$$\text{IV.} \dots \quad \Delta = i\delta_1 + j\delta_2 + k\delta_3;$$

where $\delta_1 \delta_2 \delta_3$ are three new distributive symbols, operating on functions of xyz , and commutative (in order) not only with any ordinary and scalar constants, but also with ijk .

The Secretary of the Academy presented the following donations :—

No. 1.—Captain Borrowes, Gilltown : a squared block of syenite, with a border of hieroglyphs, containing the name and title of the Egyptian king, called Sesuntesen III., by Lepsius. Dr. Todd read a letter from Dr. E. Hincks, in which he explained that the inscription contained the five titles borne by Egyptian kings, and three others, viz. :—1. “Beloved by Nû, or Noum, lord of the cataract(?)” 2. “Beloved by Sate, lady of Elephantine;” there being local deities worshipped at Elephantine. 3. “Beloved by—un (or hwn) who dwells in Nubia.” Dr. Hincks states in his note, that he never recollects to have seen the name of this deity before, and that he cannot read the hieroglyph which represents two fishes, at the commencement of this name.

Colonel Larcom explained that the interpretation of the inscription made by Dr. Hincks was most satisfactory, as he had been informed by Mr. Borrowes that he obtained the stone from a Coptic convent, situated very far up the Nile; but he hoped to get for the Academy all the particulars concerning the place and circumstances of the discovery of the stone from the donor, in a few days.

No. 2. From the Academie des Jeux Floraux, at Tou-